Streaming Erasure Codes over Multi-hop Relay Network

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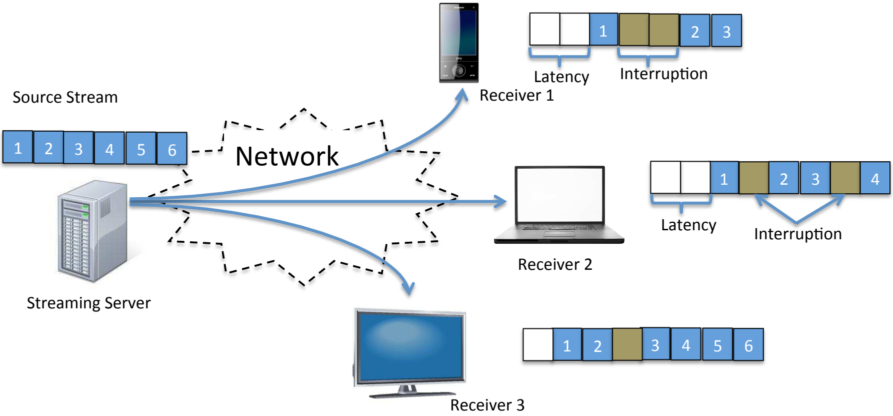
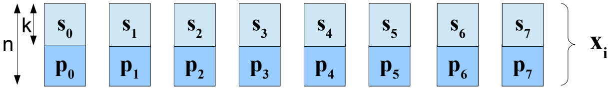
Motivation: Real-time Interactive Multimedia Streaming



FEC Streaming Codes



$$\mathbf{p}_i = \mathbf{s}_i \cdot \mathbf{H}_0 + \mathbf{s}_{i-1} \cdot \mathbf{H}_1 + \ldots + \mathbf{s}_{i-M} \cdot \mathbf{H}_M, \qquad \mathbf{s} \in \mathbb{F}^{1\times k}, \mathbf{H}_i \in \mathbb{F}^{k\times(n-k)}$$

- e.g., random linear codes, strongly-MDS codes [Gabidulin'88, Gluesing-Luerssen'06]

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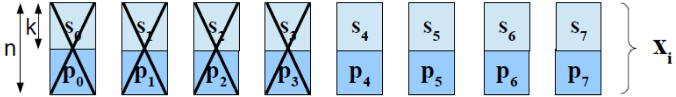

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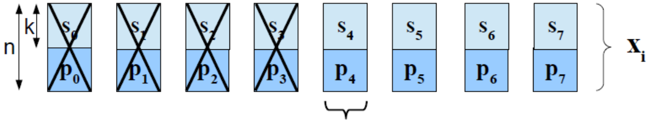

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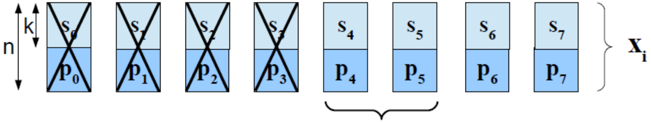

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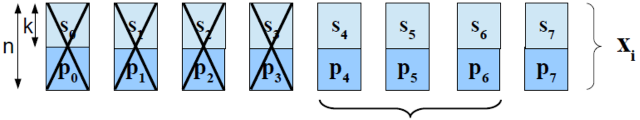

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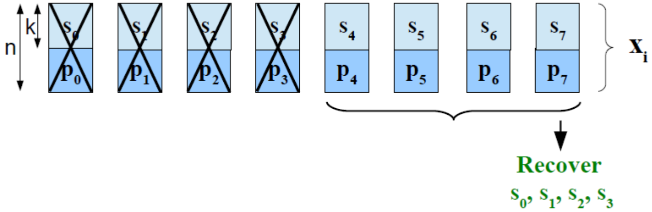

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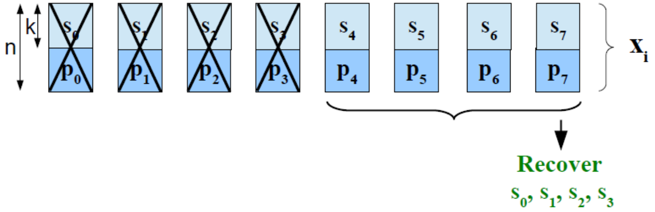
- Can correct 4 erasures

FEC Streaming Codes



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- e.g., random linear codes, strongly-MDS codes [Gabidulin'88, Gluesing-Luerssen'06]
- Can correct 4 erasures

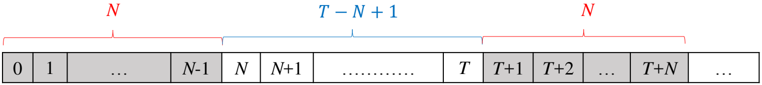
$$\begin{bmatrix}\mathbf{p}_4 & \mathbf{p}_5 & \mathbf{p}_6 & \mathbf{p}_7\end{bmatrix} = \begin{bmatrix}\mathbf{s}_0 & \mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_3\end{bmatrix} \underbrace{\begin{bmatrix}\mathbf{H}_4 & \mathbf{H}_5 & \mathbf{H}_6 & \mathbf{H}_7 \\ \mathbf{H}_3 & \mathbf{H}_4 & \mathbf{H}_5 & \mathbf{H}_6 \\ \mathbf{H}_2 & \mathbf{H}_3 & \mathbf{H}_4 & \mathbf{H}_5 \\ \mathbf{H}_1 & \mathbf{H}_2 & \mathbf{H}_3 & \mathbf{H}_4\end{bmatrix}}_{\text{full rank}}$$

Background: Point-to-Point Streaming (n, k, T)-Codes

A point-to-point channel

- A seq. of source messages: $\mathbf{s}_0, \mathbf{s}_1, \ldots$ where $\mathbf{s}_i \in \mathbb{F}^k$
- Coding rate $\frac{k}{n}$: Upon receiving $\mathbf{s}_i$, node s generates $\mathbf{x}_i \in \mathbb{F}^n$ where $\mathbf{x}_i$ is a function of $\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_i$
- A delay constraint T: Node d decodes $\mathbf{s}_i$ through outputting an estimate $\hat{\mathbf{s}}_i$ based on $\mathbf{y}_0, \mathbf{y}_1, \ldots, \mathbf{y}_{i+T}$
- To simplify analysis we assume zero propagation delay

Background: Max. Achievable Rate for (n, k, T)-Codes



A worst-case periodic erasure pattern

- Zero-error decoding: Every message $\mathbf{s}_i$ must be perfectly recovered by time $i + T$ under the (T, N)-erasure model
- (T, N)-capacity $\mathrm{C}(T, N)$: Max. coding rate k/n of an (n, k, T)-code with zero-error decoding
- We thus have: $\mathrm{C}(T, N) = \frac{k^*}{n^*} = \frac{T-N+1}{T+1}$
  - Converse: Inspect the worst-case periodic erasure pattern
  - Achievability: Periodically interleave an MDS (n^*, k^*)-block code

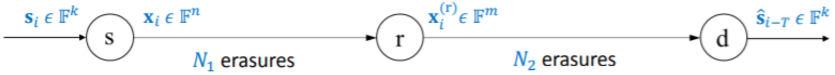
Background: Periodic Interleaving

An MDS $(5,3)$-code with $\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 4 \end{bmatrix}$ corrects $N = 2$ erasures.

Periodically interleaving $\Longrightarrow$ (n, k, T)-code with $\begin{cases} n = T + 1 = 5 \\ k = T - N + 1 = 3 \\ T = 4 \end{cases}$

	$i-2$	$i-1$	i	$i+1$	$i+2$	$i+3$	$i+4$
0	$s_{i-2}[0]$	$s_{i-1}[0]$	$s_i[0]$	$s_{i+1}[0]$	$s_{i+2}[0]$	$s_{i+3}[0]$	$s_{i+4}[0]$
1	$s_{i-2}[1]$	$s_{i-1}[1]$	$s_i[1]$	$s_{i+1}[1]$	$s_{i+2}[1]$	$s_{i+3}[1]$	$s_{i+4}[1]$
2	$s_{i-2}[2]$	$s_{i-1}[2]$	$s_i[2]$	$s_{i+1}[2]$	$s_{i+2}[2]$	$s_{i+3}[2]$	$s_{i+4}[2]$
3	$\ddots$	$\ddots$	$\ddots$	$s_{i-2}[0] + s_{i-1}[1] + s_i[2]$	$s_{i-1}[0] + s_i[1] + s_{i+1}[2]$	$s_i[0] + s_{i+1}[1] + s_{i+2}[2]$	$\ddots$
4	$\ddots$	$\ddots$	$\ddots$	$\ddots$	$s_{i-2}[0] + 2s_{i-1}[1] + 4s_i[2]$	$s_{i-1}[0] + 2s_i[1] + 4s_{i+1}[2]$	$s_i[0] + 2s_{i+1}[1] + 4s_{i+2}[2]$

Fong '19: Streaming (n, m, k, T)-Codes over Three-Node Relay Network



A three-node relay network

- A streaming message: $\mathbf{s}_0, \mathbf{s}_1, \ldots$ where $\mathbf{s}_i \in \mathbb{F}^k$
- Upon receiving $\mathbf{s}_i$, node s generates $\mathbf{x}_i \in \mathbb{F}^n$ where $\mathbf{x}_i$ is a function of $\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_i$
- Re-encoding at relay: Upon receiving $\mathbf{y}_i^{(\mathrm{r})}$, node r generates $\mathbf{x}_i^{(\mathrm{r})} \in \mathbb{F}^m$ where $\mathbf{x}_i^{(\mathrm{r})}$ is a function of $\mathbf{y}_0^{(\mathrm{r})}, \mathbf{y}_1^{(\mathrm{r})}, \ldots, \mathbf{y}_i^{(\mathrm{r})}$
- A delay constraint T: Node d decodes $\mathbf{s}_i$ through outputting an estimate $\hat{\mathbf{s}}_i$ based on $\mathbf{y}_0, \mathbf{y}_1, \ldots, \mathbf{y}_{i+T}$

(N_1, N_2)-Achievable Codes and Capacity

Definition

An (n, m, k, T)-code is said to be (N_1, N_2)-achievable if

$$\hat{\mathbf{s}}_i = \mathbf{s}_i \qquad \forall\, i \in \mathbb{Z}_+$$

Definition

Wlog, assume $T \geq N_1 + N_2$. The (T, N_1, N_2)-capacity is

$$C_{T,N_1,N_2} \triangleq \max\left\{ \frac{k}{\max\{n, m\}} \,\middle|\, \exists \text{ an } (n, m, k, T)\text{-code that is } (N_1, N_2)\text{-achievable} \right\}$$

Capacity

Theorem (Fong '19)

For any $T \geq N_1 + N_2$, we have

$$C_{T,N_1,N_2} = \min\{\mathrm{C}(T - N_2, N_1), \mathrm{C}(T - N_1, N_2)\} = \frac{T - N_1 - N_2 + 1}{T - \min\{N_1, N_2\} + 1}$$

Achievable

- Straightforward extension of point-to-point code = message-wise decode and forward
- For any $T \geq N_1 + N_2$, we have

$$\max_{T_1+T_2=T} \min\{\mathrm{C}(T_1, N_1), \mathrm{C}(T_2, N_2)\} \leq C_{T,N_1,N_2}$$

- Special case: If $T = N_1 + N_2$, achieves capacity

An Optimal Symbol-Wise DF Strategy for $N_1 = N_2 = 1$ and $T = 3$ Achieving Rate $2/3$ (Fong '19)

Time i	0	1	2	3	4
a_i	a_0	a_1	a_2	a_3	a_4
b_i	b_0	b_1	b_2	b_3	b_4
$a_{i-2} + b_{i-1}$	0	b_0	$a_0 + b_1$	$a_1 + b_2$	$a_2 + b_3$

Symbols transmitted by node s from time 0 to time 4

Time i	0	1	2	3	4	5
b_{i-1}	0	b_0	b_1	b_2	b_3	b_4
a_{i-2}	0	0	a_0	a_1	a_2	a_3
$a_{i-3} + b_{i-3}$	0	0	0	$a_0 + b_0$	$a_1 + b_1$	$a_2 + b_2$

Symbols transmitted by node r from time 0 to time 5

Time i	0	1	2	3	4	5
a_{i-3}	0	0	0	a_0	a_1	a_2
b_{i-3}	0	0	0	b_0	b_1	b_2

Symbols recovered by node d from time 0 to time 5

What do we Have so Far

- The capacity of three-node network is $\frac{T-N_1-N_2+1}{T-\min\{N_1,N_2\}+1}$
- Can be extended to a sliding window model
- **Careful modelling results in better streaming capacity:**
  - Point-to-point:

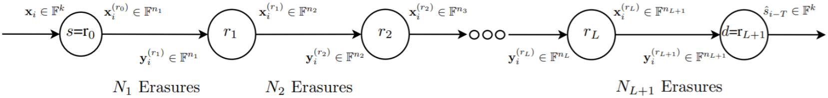
$$T=3, N=2 \implies C_{3,2}=\frac{2}{4}$$

  - Three-node network:

$$T=3, N_1=N_2=1 \implies C_{3,1,1}=\min\{C_{2,1},C_{2,1}\}=\frac{2}{3}$$

  - For any T, and any $N_1+N_2=N$, we have $C_{T,N}=\frac{T-N_1-N_2+1}{T+1} \leq \frac{T-N_1-N_2+1}{T-\min\{N_1,N_2\}+1}=C_{T,N_1,N_2}$
- In practice, the number of hops in an internet path $\gg 1$
- Can we generalize these results to any number of relays?

Streaming $(n_1, \ldots, n_{L+1}, k, T)$-Codes over L-Node Relay Network



L-node relay network

- A streaming message: $\mathbf{s}_0, \mathbf{s}_1, \ldots$ where $\mathbf{s}_i \in \mathbb{F}^k$
- Upon receiving $\mathbf{s}_i$, node s generates $\mathbf{x}_i \in \mathbb{F}^{n_1}$ where $\mathbf{x}_i$ is a function of $\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_i$
- Re-encoding at relay r_j: Upon receiving $\mathbf{y}_i^{(\mathrm{r_j})}$, node r generates $\mathbf{x}_i^{r_j\,(\mathrm{r})} \in \mathbb{F}^{n_{j+1}}$ where $\mathbf{x}_i^{r_j\,(\mathrm{r})}$ is a function of $\mathbf{y}_0^{(\mathrm{r_j})}, \mathbf{y}_1^{(\mathrm{r_j})}, \ldots, \mathbf{y}_i^{(\mathrm{r_j})}$
- A delay constraint T: Node d decodes $\mathbf{s}_i$ through outputting an estimate $\hat{\mathbf{s}}_i$ based on $\mathbf{y}_0^{r_{L+1}}, \mathbf{y}_1^{r_{L+1}}, \ldots, \mathbf{y}_{i+T}^{r_{L+1}}$

$(N_1, \ldots, N_{L+1})$-Achievable Codes and Capacity

Definition

An $(n_1, \ldots, n_{L+1}, k, T)$-code is said to be $(N_1, \ldots, N_{L+1})$-achievable if

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Definition

Wlog, assume $T \geq \sum_{l=1}^{L+1} N_l$. The $(T, N_1, \ldots, N_{L+1})$-capacity is

$$C_{T,N_1,\ldots,N_{L+1}} \triangleq \max \left\{ \frac{k}{\max\{n_1, \ldots, n_{L+1}\}} \;\middle|\; \exists \text{ an } (n_1, \ldots, n_{L+1}, k, T)\text{-code that is } (N_1, \ldots, N_{L+1})\text{-achievable} \right\}$$

Theorem (converse)

In an L+1-node network with maximum of N_j erasures in link (r_{j-1}, r_j), when $T \geq \sum_{l=1}^{L+1} N_l$ the streaming rate us upper bounded by

$$R \leq \frac{T - \sum_{l=1}^{L+1} N_l + 1}{T - \min_j \sum_{l=1, l \neq j}^{L+1} N_l + 1} = \min C_{T - \sum_{l=1, l \neq j}^{L+1} N_l, N_j}$$

$$\triangleq C^{+}_{T, N_1, \ldots, N_{L+1}}$$

Theorem (achievable)

Theorem: In an L+1-node network with maximum of N_j erasures in link (r_{j-1}, r_j), when $T \geq \sum_{l=1}^{L+1} N_l$ the following streaming rate is achievable

$$R \geq \min_j \frac{k \cdot |\mathbb{F}|}{n_{j,j+1} \cdot |\mathbb{F}| + n_{\max} \lceil \log(n_{\max}) \rceil}$$

$$= \frac{T - \sum_{l=1}^{L+1} N_l + 1}{\max_j \left\{ T - \sum_{l=1, l \neq j}^{L+1} N_l + 1 \right\} + \frac{n_{\max} \lceil \log(n_{\max}) \rceil}{|\mathbb{F}|}}$$

Achievable Scheme

- Three-node network: $C_{3,1,1} = \frac{2}{3}$, four-node network: $C_{4,1,1,1} \leq \frac{2}{3}$
- Can we extend the three-node scheme which achieves capacity?

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- Can we extend the three-node scheme which achieves capacity?

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a_i	a_0	a_1	a_2	a_3	a_4
b_i	b_0	b_1	b_2	b_3	b_4
$a_{i-2} + b_{i-1}$	0	b_0	$a_0 + b_1$	$a_1 + b_2$	$a_2 + b_3$

Symbols transmitted by node s from time 0 to time 4

Time i	0	1	2	3	4	5
b_{i-1}	0	b_0	b_1	b_2	b_3	b_4
a_{i-2}	0	0	a_0	a_1	a_2	a_3
$a_{i-3} + b_{i-3}$	0	0	0	$a_0 + b_0$	$a_1 + b_1$	$a_2 + b_2$

Symbols transmitted by node r from time 0 to time 5

Time i	0	1	2	3	4	5
a_{i-3}	0	0	0	a_0	a_1	a_2
b_{i-3}	0	0	0	b_0	b_1	b_2

Symbols recovered by node d from time 0 to time 5

- Destination $\Longrightarrow$ relay r_2. Note: all delays are "reset" at r_2
- Can achieve rate $2/3$ for $N_1 = N_2 = N_3 = 1$ and $T = 5$

Another Strategy for Achieving $C_{3,1,1}$

Time i	0	1	2	3	4
a_i	a_0	a_1	a_2	a_3	a_4
b_i	b_0	b_1	b_2	b_3	b_4
$a_{i-2} + b_{i-1}$	0	b_0	$a_0 + b_1$	$a_1 + b_2$	$a_2 + b_3$

Symbols transmitted by node s from time 0 to time 4

Time i	0	1	2	3	4	5
a_{i-1}	0	a_0	a_1	a_2	a_3	a_4
b_{i-1}	0	b_0	b_1	b_2	b_3	b_4
$a_{i-3} + b_{i-2}$	0	0	b_0	$a_0 + b_1$	$a_1 + b_2$	$a_2 + b_3$

Symbols transmitted by node r from time 0 to time 5

Time i	0	1	2	3	4	5
a_{i-3}	0	0	0	a_0	a_1	a_2
b_{i-3}	0	0	0	b_0	b_1	b_2

Symbols recovered by node d from time 0 to time 5

- Too good to be true... what happens if there are erasures?

Another Strategy for Achieving $C_{3,1,1}$

Time i	0	1	2	3	4
a_i	a_0	a_1	a_2	a_3	a_4
b_i	b_0	b_1	b_2	b_3	b_4
$a_{i-2}+b_{i-1}$	0	b_0	a_0+b_1	a_1+b_2	a_2+b_3

Symbols transmitted by node s from time 0 to time 4

Time i	0	1	2	3	4	5
a_{i-1}	0	a_0	a_1	a_2	a_3	a_4
b_{i-1}	0	b_0	b_1	b_2	b_3	b_4
$a_{i-3}+b_{i-2}$	0	0	b_0	a_0+b_1	a_1+b_2	a_2+b_3

Symbols transmitted by node r from time 0 to time 5

Time i	0	1	2	3	4	5
a_{i-3}	0	0	0	a_0	a_1	a_2
b_{i-3}	0	0	0	b_0	b_1	b_2

Symbols recovered by node d from time 0 to time 5

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Time i	0	1	2	3	4
a_i	a_0	a_1	a_2	a_3	a_4
b_i	b_0	b_1	b_2	b_3	b_4
$a_{i-2} + b_{i-1}$	0	b_0	$a_0 + b_1$	$a_1 + b_2$	$a_2 + b_3$

Symbols transmitted by node s from time 0 to time 4

Time i	0	1	2	3	4	5
a_{i-1}	0	b_1	a_1	a_2	a_3	a_4
b_{i-1}	0	b_0	a_0	b_2	b_3	b_4
$a_{i-3} + b_{i-2}$	0	0	b_0	$a_0 + b_1$	$a_1 + b_2$	$a_2 + b_3$

Symbols transmitted by node r from time 0 to time 5

Time i	0	1	2	3	4	5
a_{i-3}	0	0	0	a_0	a_1	a_2
b_{i-3}	0	0	0	b_0	b_1	b_2

Symbols recovered by node d from time 0 to time 5

- Too good to be true... what happens if there are erasures?
- **Key observation:** per hop (per diagonal), there are always “enough” symbols to send (not necessarily in the right order...)

Achievable scheme for Multi-hop Network

- Denote $\tilde{\mathbf{s}}_i = [s_i[0]\ s_{i+1}[1]\ \cdots\ s_{i+k-1}[k-1]]$
- Sender: Encode $\tilde{\mathbf{s}}_i$ using an $(n_1, k) = (T - \sum_{l=2}^{L+1} N_l + 1, T - \sum_{l=1}^{L+1} N_l + 1)$ MDS block code and transmit it over the diagonal starting at time i
- Relay r_1:
  - Start transmitting $\tilde{\mathbf{s}}_i$ at time $i + N_1$.
  - Until time $i + T - \sum_{l=2}^{L+1} N_l - 1 = i + n_1 - 2$ forward any received symbols at the order they were received
  - At time $i + T - \sum_{l=2}^{L+1} N_l = i + n_1 - 1$ decode $\tilde{\mathbf{s}}_i$ and encode it such that the combination of forwarded and encoded symbols results in an $(n_2, k) = (T - \sum_{l=1, l\neq 2}^{L+1} N_l + 1, T - \sum_{l=1}^{L+1} N_l + 1)$ MDS block code
- Relay r_j - generalize the coding scheme of r_1

Summary of the Suggested Strategy

- Transmission is no longer "state-independent" (i.e., is not independent of the erasure pattern on previous links)
- However, this scheme can be easily extended to any number of relays
- Challenges:
  - The order of symbols in each code may depend on erasure patterns in previous hops
  - Can we guarantee that an MDS code can be generated from any combination of $k-1$ received symbols from previous node?
- Suggested solutions:
  - Add a header that will indicate the order of symbols
  - Proposition described next

Bounding the Size of the Header

Proposition

All block codes used by nodes $j \in [0, \dots, L]$ can be generated by puncturing the MDS code associated with rate $C^{+}_{T,N_1,\dots,N_{L+1}}$

- All codes can be viewed as sub-codes of the $(n_{\max}, k)$ MDS code.
- Worst case: the header consists of $n_{\max}$ elements, each one chosen from $[1, \dots, n_{\max}]$
- Repetitions are allowed per packet
- The size of the header is upper bounded by $n_{\max}\lceil\log(n_{\max})\rceil$

Achieving Rate $2/3$ for $N_1 = N_2 = N_3 = 1$ and $T = 4$

Time i	0	1	2	3	4
a_i	a_0	a_1	a_2	a_3	a_4
b_i	b_0	b_1	b_2	b_3	b_4
$a_{i-2} + b_{i-1}$	0	b_0	$a_0 + b_1$	$a_1 + b_2$	$a_2 + b_3$

Symbols transmitted by node s from time 0 to time 4

Time i	0	1	2	3	4	5
Header	123	223	113	123	123	123
a_{i-1}	0	b_1	a_1	a_2	a_3	a_4
b_{i-1}	0	b_0	a_0	b_2	b_3	b_4
$a_{i-3} + b_{i-2}$	0	0	b_0	$a_0 + b_1$	$a_1 + b_2$	$a_2 + b_3$

Symbols transmitted by node r_1 from time 0 to time 5

Time i	0	1	2	3	4	5
Header	123	123	223	213	113	123
a_{i-1}	0	0	b_1	b_2	a_2	a_3
b_{i-1}	0	0	b_0	a_0	a_1	b_3
$a_{i-3} + b_{i-2}$	0	0	0	b_0	$a_0 + b_1$	$a_1 + b_2$

Symbols transmitted by node r_2 from time 0 to time 5

Summary and Future work

Summary

- For a general setting with any number of relays we showed
  - Upper bound
  - A symbol-wise DF scheme which depends on the erasure pattern
- The gap of the achievable scheme from the upper bound vanishes as $|\mathbb{F}|$ increases.

Future work

- Can we find a scheme which achieves the upper bound without the need for a header?
- Can we extend this analysis to relay setting with burst losses?
- Implement these codes in real-life setups